

Comment on "Spin Contribution to the Ponderomotive Force in a Plasma"

Pavel A. Andreev

andreevpa@physics.msu.ru

Lomonosov Moscow State University, Moscow, Russia.

Several years ago, G. Brodin, A. P. Misra, and M. Marklund considered "Spin Contribution to the Ponderomotive Force in a Plasma" [1]. They applied a two fluid model of electron gas where spin-up and spin-down electrons are considered as two different species. For each species of electrons they used the continuity equations, the Euler equations and the spin evolution equations coinciding with the equations obtained for single fluid description of electrons. This approach, however, appears to be incorrect.

Single fluid quantum model of electrons results from the Pauli equation [2], or, more precisely, from the many-particle Pauli equation [3], [4].

The single particle Pauli equation presents the evolution of the two-component spinor wave function ψ describing the probability of the electron to have a spin directed "up" or "down" (parallel or anti-parallel to the external magnetic field). A similar, but rather more complicate, situation can be observed in many-particle systems of spin-1/2 particles. The fluidisation of the many-particle model [3] leads to a model which, under the self-consistent field approximation and with no explicit account of thermal evolution, such as time evolution of the energy density or pressure, reveals in a set of equations similar to the single particle case [2]. Thus, if we want to capture the main properties of the two-fluid (separated spin-up and spin-down) model of electrons we can use the single particle Pauli equation.

If we want to trace the evolution of spin-up and spin-down electrons separately we should consider ψ_u and ψ_d separately as well. Each of the one component wave functions defines the concentration of electrons $n_u = \psi_u^* \psi_u$ and $n_d = \psi_d^* \psi_d$, whereas the full concentration $n = n_u + n_d = \psi^+ \psi$. Starting with these definitions of concentrations for spin-up and spin-down electrons we can perform a two-fluid fluidization of the Pauli equation.

The numbers of spin-up and spin-down electrons are conserved in the absence of the spin-spin interaction. The spin-spin interaction leads to the nontrivial evolution of the spin densities S_x and S_y . In this comment we do not consider the effects related to the nonconservation of numbers of spin-up and spin-down electrons.

While the z-projection of spin density S_z of electrons is not an independent variable in this model, S_z appears

as the difference between the concentrations of electrons with different projections of spin $S_z = n_u - n_d$ due to its definition $S_z = \psi^+ \sigma_z \psi$.

The spin-up and spin-down directions are related to a preferable direction in space. If we have a strong uniform external magnetic field, its direction can be taken as a preferable direction. This field reveals in z-projection of the magnetic field B_z in the Pauli equation. However, the propagation of an electromagnetic wave parallel to z axis creates B_x and B_y as well.

Considering the time evolution of the probability densities n_u and n_d we derive the continuity equations $\partial_t n_s + \nabla \cdot (n_s \mathbf{v}_s) = 0$. They show the conservation of the particle number. The Euler equation arises as

$$(\partial_t + \mathbf{v}_s \cdot \nabla) \mathbf{v}_s + \frac{\nabla p_s}{m n_s} - \frac{\hbar^2}{2m^2} \nabla \left(\frac{\Delta \sqrt{n_s}}{\sqrt{n_s}} \right) = \frac{q_e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_s, \mathbf{B}] \right) + (-1)^{i_s} \frac{\mu}{m} \nabla B_z + \frac{\mu}{2m n_s} (S_x \nabla B_x + S_y \nabla B_y), \quad (1)$$

where $q_e = -e$, $\mu = -\mu_B$, with $\mu_B = e\hbar/(2mc)$ being the Bohr magneton, and $i_u = 0$, $i_d = 1$.

We have used the notations $S_x = \psi^* \sigma_x \psi$ and $S_y = \psi^* \sigma_y \psi$ in the equation (1). S_x and S_y appear as mixed combinations of ψ_u and ψ_d . S_x and S_y describe the simultaneous evolution of both species and do not wear subindexes u and d . The spin evolution is described by $\partial_t S^\alpha = \frac{2\mu}{\hbar} \varepsilon^{\alpha\beta\gamma} S^\beta B^\gamma$, where α stands for x, y , whereas $\beta, \gamma = x, y, z$, and $S_z = n_u - n_d$.

We found that the ponderomotive forces acting on spin-up and spin-down electrons

$$\mathbf{F}_s = \pm \frac{\mu^2}{2\hbar} \frac{n_u - n_d}{n_s} \frac{\partial_z |B_\pm|^2}{\omega \mp \Omega} \quad (2)$$

have the same sign for the spin-up and the spin-down electrons. This force depends on the polarisation of the light and the rate of the spin polarisation of the medium. The module of the force is found to be four times smaller than the result of the Ref. [1].

We conclude that Brodin et al. [1] started with the incorrect model of the separate description of the spin-up and the spin-down electrons. As a result, they obtained the incorrect generalisation for the ponderomotive force. The correct model and the correct expression for the ponderomotive force of the zeroth order are presented in this comment.

[1] G. Brodin, A. P. Misra, and M. Marklund, Phys. Rev. Lett. **105**, 105004 (2010).

[2] T. Takabayasi, Prog. Theor. Phys. **14**, 283 (1955).

[3] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev,

Theoretical and Mathematical Physics, **126**, 110 (2001).

[4] P. A. Andreev, L. S. Kuz'menkov, Russ. Phys. J. **50**, 1251 (2007).